

Manipulating and probing Majorana fermions using superconducting circuits

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Majorana fermions are long-sought exotic particles that are their own antiparticles and satisfy non-Abelian statistics. Here we utilize superconducting circuits to construct two superconducting-qubit arrays where Majorana modes can occur. Moreover, we propose to use four superconducting qubits as the smallest system to demonstrate the braiding of Majorana modes and show how the states before and after braiding Majoranas can be discriminated. This provides an experimentally realizable, relatively simple system for manipulating and probing Majorana fermions.

Introduction.—Majorana fermions are particles that are their own antiparticles and satisfy non-Abelian statistics. These long-sought particles have recently received considerable interest (see, e.g., [1–8]). It has been recognized [9] that a relatively easy to engineer system—one-dimensional (1D) semiconducting wires on an s -wave superconductor—can realize a nontrivial topological state supporting Majorana fermions. This state is characteristic of 1D topological superconductors [4], in which Majorana modes can occur without requiring the presence of vortices in the system. Moreover, it was proposed [10] to use tunable 1D semiconducting wire networks on an s -wave superconductor to demonstrate the non-Abelian statistics of Majorana fermions, because the Majoranas in the semiconducting wires can also behave like vortices in a $p + ip$ superconductor [2, 3]. In addition, it was also recognized [11] that when a Jordan-Wigner transformation is performed, a 1D quantum Ising model is equivalent to a 1D topological superconductor, and Majorana modes can also occur therein. Nevertheless, less attention has been paid to this quantum Ising model than to 1D topological superconductors because it was often regarded as a toy model.

In this Letter, we unambiguously show that such a toy model can be realized experimentally by using superconducting-qubit arrays. Importantly, superconducting qubits can behave as controllable artificial atoms and tunable interqubit couplings are also achievable (see, e.g., [12]). These distinct advantages of superconducting qubits make it possible to construct a tunable 1D quantum Ising model on wire networks, similar to the semiconducting wire networks in [10], to demonstrate the non-Abelian statistics of Majorana fermions. We propose to use four superconducting qubits as the smallest circuit to demonstrate the braiding of Majorana modes, and show how the states before and after braiding Majoranas can be discriminated. This should provide an experimentally realizable, relatively simple setup to manipulate and probe Majorana fermions.

Majorana fermions in superconducting circuits.—We construct two types of superconducting-qubit arrays (see Fig. 1), which can exhibit Majorana modes.

(i) *Charge-qubit array.* For the array of charge qubits shown in Fig. 1(a), every pair of nearest-neighbor qubits are coupled by a large Josephson junction acting as an effective inductance. The non-nearest-neighbor qubits can also be coupled via these large Josephson junctions, but the interactions are negligibly small. Here we assume that all charge qubits are identical and that all large junctions are equal to each other. When leading terms are considered, the Hamiltonian of the charge-qubit array can be written as

$$H = \sum_{n=1}^{N-1} t \sigma_n^x \sigma_{n+1}^x - \sum_{n=1}^N (\mu \sigma_n^z + E_J \sigma_n^x), \quad (1)$$

with $\mu = \frac{1}{2} E_{\text{ch}} (1 - C_g V_g / e)$, $E_J(\Phi_q) = E_{J0} \cos(\pi \Phi_q / \Phi_0)$, and the interqubit coupling is given by [13]

$$t = L_J \left(\frac{\pi E_{J0}}{\Phi_0} \right)^2 \sin^2 \left(\frac{\pi \Phi_q}{\Phi_0} \right). \quad (2)$$

Here, $E_{\text{ch}} (\approx e^2 / C_J) \gg E_J$, in the considered charging regime and $L_J = \Phi_0 / 2\pi I_c$, with $I_c = 2\pi E_{Jc} / \Phi_0$ and Φ_0 being the flux quantum. The eigenstates of the Pauli operator σ_n^z are the charge states $|0_n\rangle$ and $|1_n\rangle$, corresponding to zero and one extra Cooper pair in the superconducting island of the n th qubit. The Hamiltonian (1) provides an analog to the 1D quantum Ising model.

We now consider the case with the fluxes in all charge-qubit loops being tuned to $\Phi_q = \frac{1}{2} \Phi_0$, so that $E_J(\Phi_q) = 0$, and the interqubit couplings reach the maximum $t = L_J (\pi E_{J0} / \Phi_0)^2$. Using the Jordan-Wigner transformation: $a_n = \sigma_n^- \prod_{m=1}^{n-1} \sigma_m^z$, $a_n^\dagger = \sigma_n^+ \prod_{m=1}^{n-1} \sigma_m^z$, where $\sigma_n^\pm = \frac{1}{2} (\sigma_n^x \pm i \sigma_n^y)$, one can cast Eq. (1), in the case of $\Phi_q = \frac{1}{2} \Phi_0$, to

$$H = \sum_{n=1}^{N-1} t (a_n - a_n^\dagger)(a_{n+1} + a_{n+1}^\dagger) - \sum_{n=1}^N \mu (2a_n^\dagger a_n - 1), \quad (3)$$

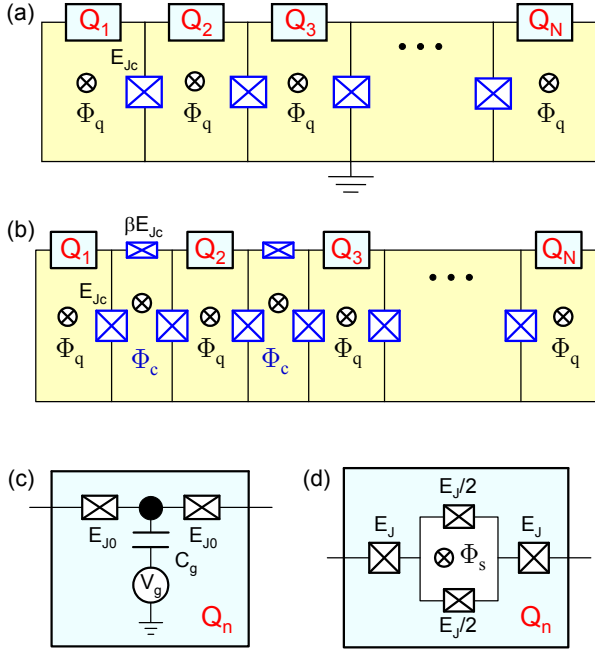


FIG. 1: (Color online) Two arrays of superconducting qubits. (a) Charge-qubit array: Nearest-neighbor charge qubits Q_n and Q_{n+1} are coupled by a large Josephson junction with coupling energy E_{Jc} (shown as a crossed rectangle). (b) Flux-qubit array: Nearest-neighbor flux qubits are coupled by a coupler consisting of a flux-biased loop that is interrupted by two large Josephson junctions (each with coupling energy E_{Jc}) and a small Josephson junction with coupling energy βE_{Jc} , where $0 < \beta \ll 1$. In (a) and (b), Φ_q is the flux applied to each qubit loop. (c) Main components of a charge qubit, where a superconducting island (denoted as a solid circle) is connected to two Josephson junctions (each with coupling energy $E_{J0} \ll E_{Jc}$ and capacitance C_J) and biased by a voltage V_g through a gate capacitance $C_g \ll C_J$. (d) Main components of a flux qubit, where two Josephson junctions, each with coupling energy $E_J \ll E_{Jc}$, connects a symmetric dc SQUID biased by a flux Φ_s .

where the Dirac fermions obey the anticommutation relation $\{a_n, a_{n'}^\dagger\} = \delta_{nn'}$. Introducing Majorana fermions:

$$\gamma_n^A = a_n^\dagger + a_n, \quad \gamma_n^B = i(a_n^\dagger - a_n), \quad (4)$$

one can rewrite the Hamiltonian (3) as

$$H = i \sum_{n=1}^{N-1} t \gamma_n^B \gamma_n^A - i \sum_{n=1}^N \mu \gamma_n^A \gamma_n^B, \quad (5)$$

where $\gamma_n^{X\dagger} = \gamma_n^X$ and $\{\gamma_n^X, \gamma_{n'}^{X'}\} = 2\delta_{XX'}\delta_{nn'}$.

(ii) *Flux-qubit array.* Figure 1(b) shows an array of flux qubits. Here the small junction in the ordinary flux qubit is replaced by a symmetric dc SQUID to increase the tunability of the qubit. Also, a coupler consisting of three Josephson junctions is used to produce a controllable interqubit coupling between nearest-neighbor flux qubits. We assume that the parameters are the same

for all qubits and also for all couplers. Moreover, the plasma frequency ν_p of the coupler is much higher than the related qubit energy, so as to have the coupler in the ground state [14]. When the leading terms are included, the Hamiltonian of the flux-qubit array can be written as

$$H = \sum_{n=1}^{N-1} t \sigma_n^z \sigma_{n+1}^z - \sum_{n=1}^N (\varepsilon \sigma_n^z + \mu \sigma_n^x). \quad (6)$$

Here $\varepsilon = I_p \Phi_0 (\frac{1}{2} - f)$, with I_p being the persistent current of the flux qubit and $f = \Phi_q / \Phi_0 + f_s / 2$, where $f_s = \Phi_s / \Phi_0$. The eigenstates of the Pauli operator σ_n^z are the clockwise and anti-clockwise persistent-current states of the n th qubit. The symmetric SQUID provides an effective Josephson junction with coupling energy αE_J , where $\alpha = |\cos(\pi f_s)|$. The exact expression of μ in Eq. (6) cannot be given, but it depends on α ; numerical results [15] and approximate analytical calculations [16] showed that $\mu = 0$ when $\alpha = 1$. The interqubit coupling strength reads [14]

$$t = \frac{\beta E_{Jc} \cos(2\pi f_c - \phi_c)}{1 + 2\beta \cos(2\pi f_c - \phi_c)}, \quad (7)$$

where $\phi_c = 2\beta \sin(2\pi f_c) / [1 + 2\beta \cos(2\pi f_c)]$, and $f_c = \Phi_c / \Phi_0$ is the reduced flux applied to the coupler.

We study the case with $f = \frac{1}{2}$ for all flux qubits, so as to have $\varepsilon = 0$. The Hamiltonian of the system also becomes Eq. (3) when applying the Jordan-Wigner transformation: $a_n = \sigma_n^+ \prod_{m=1}^{n-1} \sigma_m^x$, $a_n^\dagger = \sigma_n^- \prod_{m=1}^{n-1} \sigma_m^x$, where $\sigma_n^\pm = \frac{1}{2}(\sigma_n^x \pm i\sigma_n^y)$. Finally, the Hamiltonian is described by Eq. (5) by introducing Majorana fermions in Eq. (4). Therefore, the resulting Hamiltonians in terms of Majorana fermions are the same for both charge- and flux-qubit arrays.

For $N \rightarrow \infty$, we can obtain the energy bands of the periodic chain by performing a Fourier transform on Hamiltonian (5): $\gamma_n^X = \sqrt{\frac{2}{N}} \sum_k e^{ikn} \gamma_k^X$, where $\gamma_k^X = \gamma_k^{X\dagger}$ and $X = A, B$. The resulting Hamiltonian in the reciprocal space reads

$$H = \sum_k \begin{pmatrix} \gamma_k^{A\dagger} & \gamma_k^{B\dagger} \end{pmatrix} \begin{pmatrix} 0 & -iD^*(k) \\ iD(k) & 0 \end{pmatrix} \begin{pmatrix} \gamma_k^A \\ \gamma_k^B \end{pmatrix}, \quad (8)$$

with $D(k) = te^{ik} + \mu$. The energy spectrum shows the particle-hole symmetric dispersion $E(k) = \pm |D(k)| = \pm |te^{ik} + \mu|$, which consists of two bands.

When $|r| < 1$, where $r \equiv \mu/t$, there are two degenerate edge modes with zero energy (i.e., in the middle of the energy gap) for a finite chain. These two edge modes can be represented by $Q = c_1 \gamma_1^A + c_2 \gamma_1^B + \dots + c_{2N-1} \gamma_N^A + c_{2N} \gamma_N^B$, with the coefficients determined by $\mu c_{2n-1} + t c_{2n+1} = 0$, and $t c_{2n-2} + \mu c_{2n} = 0$, where $n = 1, 2, \dots, N$, and the initial condition is $c_0 = 0$ for the left-end edge state and $c_{2N+1} = 0$ for the right-end edge state. It

can be derived that the left-end edge mode is given by $Q_L = C[\gamma_1^A - r\gamma_2^A + r^2\gamma_3^A - \dots + (-r)^{N-1}\gamma_N^A]$, and the right-end edge mode becomes $Q_R = C[(-r)^{N-1}\gamma_1^B + \dots + r^2\gamma_{N-2}^B - r\gamma_{N-1}^B + \gamma_N^B]$, where the normalization factor is $C = (\sum_{n=0}^{N-1} r^{2n})^{-1/2}$. In particular, when $\mu = 0$, the Hamiltonian is reduced to $H = -\sum_{n=1}^{N-1} t(2d_n^\dagger d_n - 1)$, where $d_n = \frac{1}{2}(\gamma_{n+1}^A + i\gamma_n^B)$ is a Dirac fermion composed of two Majoranas at adjoining sites. The edge modes become two isolated Majorana fermions at the left and right ends of the chain: $Q_L = \gamma_1^A$, and $Q_R = \gamma_N^B$, which do not appear in the Hamiltonian because these modes have zero energy [4, 17]. Now define $|F\rangle$ to be the state in which all eigenstates of the system with $E < 0$ are occupied and those with $E \geq 0$ are empty. When the edge modes are occupied, $|\Psi_L\rangle = \gamma_1^A|F\rangle$ and $|\Psi_R\rangle = \gamma_N^B|F\rangle$ are two degenerate ground states of the system. These two Majorana modes can be used to represent the states of a topologically-protected logical qubit [10]: $|0\rangle \equiv d_{\text{end}}|F\rangle$ and $|1\rangle \equiv d_{\text{end}}^\dagger|0\rangle$, where $d_{\text{end}} = \frac{1}{2}(\gamma_1^A + i\gamma_N^B)$ is a non-local Dirac fermion, and $d_{\text{end}}|0\rangle = 0$.

Manipulating and probing Majorana fermions.— The superconducting-qubit arrays proposed above can be used to realize a tunable 1D quantum Ising model on wire networks, similar to the semiconducting wire networks in [10], to demonstrate the non-Abelian statistics of Majorana fermions. In particular, braiding Majoranas can be implemented via a T-junction formed by two perpendicular wires [10]. Here we use four superconducting qubits, as the smallest size of the system, to form such a T-junction [see Fig. 2 (a)], where $E_J = 0$ ($\varepsilon = 0$) for all charge (flux) qubits. When the Jordan-Wigner transformation is performed, this T-junction of four qubits is described by

$$\begin{aligned}
H = & t(a_1 - a_1^\dagger)(a_2^\dagger + a_2) + t(a_{1'} - a_{1'}^\dagger)(a_2^\dagger + a_2) \\
& + t(a_2 - a_2^\dagger)(a_3^\dagger + a_3) - \mu(2a_1^\dagger a_1 - 1) \\
& - \mu(2a_{1'}^\dagger a_{1'} - 1) - \mu(2a_2^\dagger a_2 - 1) - \mu(2a_3^\dagger a_3 - 1) \\
= & i(t\gamma_1^B \gamma_2^A + t\gamma_{1'}^B \gamma_2^A + t\gamma_2^B \gamma_3^A) \\
& - i(\mu\gamma_1^A \gamma_1^B + \mu\gamma_{1'}^A \gamma_{1'}^B + \mu\gamma_2^A \gamma_2^B + \mu\gamma_3^A \gamma_3^B), \quad (9)
\end{aligned}$$

where qubits are numbered by starting from sites 1 and 1' and ending at site 3.

For Hamiltonian (9), when $t = 0$ for all pairs of nearest-neighbor qubits, no edge states occur and the whole T-junction region is non-topological. Starting from this phase, we adiabatically vary the parameters of superconducting qubits to have the horizontal array become a topological region, i.e., the Hamiltonian of the system becomes $H = i(t\gamma_1^B \gamma_2^A + t\gamma_2^B \gamma_3^A) - i\mu\gamma_1^A \gamma_1^B = -t(2a_1^\dagger d_1 - 1) - t(2d_2^\dagger d_2 - 1) - \mu(2a_{1'}^\dagger a_{1'} - 1)$. This creates a pair of Majoranas at the two ends of the horizontal array [see Fig. 2(b)]. Generally, the system takes the superposition state of these two degenerate Majorana modes: $|\Psi\rangle = (a\gamma_1^A + b\gamma_3^B)|F\rangle$, where $|F\rangle$ is the state in which all eigenstates of the system with $E < 0$ are occupied. How-

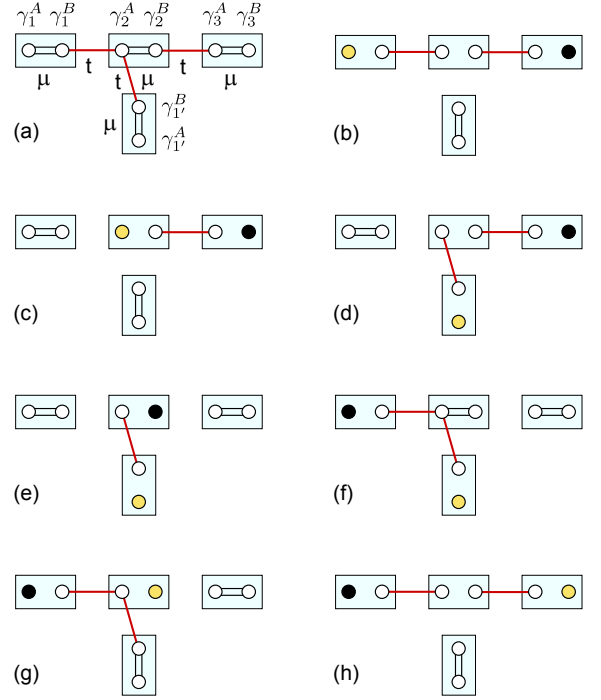


FIG. 2: (Color online) Braiding two unpaired Majorana fermions. (a) T-junction formed by qubits 1, 2, 3 and 1', where each qubit is denoted by a rectangular box. Two Majoranas related to the same qubit (e.g., γ_1^A and γ_1^B) can be paired by the parameter μ while two Majoranas related to adjoining qubits (e.g., γ_1^B and γ_2^A) can be paired by t . (b) Topological region for the whole horizontal array, where a unpaired Majorana (denoted as a solid circle) occurs at each end. (c) Adiabatically tuning μ to nonzero for qubit 1 and turning off t between qubits 1 and 2 drive the left-end Majorana mode (shown in yellow) to the middle qubit. (d) Adiabatically tuning μ to zero for qubit 1' and turning on t between qubits 2 and 1' drive the original left-end Majorana to the bottom of the T-junction. (e) The right-end Majorana (in black) is driven to the middle qubit by adiabatically tuning μ to nonzero for qubit 3 and turning off t between qubits 2 and 3. (f) The original right-end Majorana is finally driven to the left end by adiabatically tuning μ to a sufficiently large value for qubit 2, tuning μ to zero for qubit 1 and turning on t between qubits 1 and 2. (g) The Majorana at the bottom is driven to the middle qubit by adiabatically tuning μ to zero for qubit 2 and tuning μ to a sufficiently large value for qubit 1'. (h) Adiabatically turning off t between qubits 2 and 1', tuning μ to zero for qubit 3, and turning on t between qubits 2 and 3 finally drive the original left-end Majorana to the right end. This accomplishes the anti-clockwise braiding of two Majorana fermions.

ever, while reaching the state in Fig. 2(b), if μ for qubits 1, 2 and 3 are all adiabatically tuned to zero in the same manner and the interqubit coupling between qubits 1 and 2 is adiabatically switched on in the same way as that between qubits 2 and 3, then the left- and right-end Majoranas should occur with equal probabilities. Using this state $|\Psi\rangle_i = \frac{1}{2}(\gamma_1^A + e^{i\theta}\gamma_3^B)|F\rangle$ as the initial state, one can

braid the left- and right-end Majoranas through the steps shown in Figs. 2(c)-2(h) by adiabatically tuning the qubit parameters. This braiding of Majoranas corresponds to a unitary operator [3, 10] $U = \exp(\pi\gamma_3^B\gamma_1^A/4)$, which transforms γ_1^A to $U\gamma_1^AU^{-1} = \gamma_3^B$ and γ_3^B to $U\gamma_3^BU^{-1} = -\gamma_1^A$. Therefore, the initial state $|\Psi\rangle_i$ of the system is transferred to $|\Psi\rangle_f = \frac{1}{2}(\gamma_3^B - e^{i\theta}\gamma_1^A)|F\rangle$ after braiding the left- and right-end Majoranas.

Finally, we focus on probing Majorana fermions. The initial state $|\Psi\rangle_i = \frac{1}{2}(\gamma_1^A + e^{i\theta}\gamma_3^B)|F\rangle$ given in Fig. 2(b) is a ground state of the system with qubit 1' decoupled from the horizontal array of superconducting qubits. When expressed in the basis states of qubits, this initial state can be written as $|\Psi\rangle_i = |\Psi_{123}\rangle_i \otimes |\Psi_{1'}\rangle_i$, where $|\Psi_{123}\rangle_i = \lambda_{i1}|0_10_20_3\rangle + \lambda_{i2}|0_10_21_3\rangle + \lambda_{i3}|0_11_20_3\rangle + \lambda_{i4}|0_11_21_3\rangle + \lambda_{i5}|1_10_20_3\rangle + \lambda_{i6}|1_10_21_3\rangle + \lambda_{i7}|1_11_20_3\rangle + \lambda_{i8}|1_11_21_3\rangle$, and $|\Psi_{1'}\rangle_i = \xi_{i1}|0_{1'}\rangle + \xi_{i2}|1_{1'}\rangle$. Also, the final state $|\Psi\rangle_f = \frac{1}{2}(\gamma_3^B - e^{i\theta}\gamma_1^A)|F\rangle$ is another degenerate ground state of the same system and can be expressed as $|\Psi\rangle_f = |\Psi_{123}\rangle_f \otimes |\Psi_{1'}\rangle_f$, where $|\Psi_{1'}\rangle_f = \xi_{f1}|0_{1'}\rangle + \xi_{f2}|1_{1'}\rangle$, and $|\Psi_{123}\rangle_f$ has the same form as $|\Psi_{123}\rangle_i$, but the λ_{il} are replaced by λ_{fl} , with $l = 1$ to 8. The states $|\Psi\rangle_i$ and $|\Psi\rangle_f$ can be distinguished using experimentally available state-tomography techniques for superconducting qubits (see, e.g., [18, 19]), which involve reconstructing an unknown quantum state from a complete set of measurements of the system observables.

Experimentally, it is more complicated to use state-tomography techniques to determine the quantum state of three qubits other than two qubits. Therefore, we can consider the state $|\bar{\Psi}\rangle_i$ in Fig. 2(c) as the initial state. This state is a ground state of the system with qubits 1' and 1 decoupled from other qubits and can be decomposed as $|\bar{\Psi}\rangle_i = |\Psi_{23}\rangle_i \otimes |\Psi_{1'}\rangle_i \otimes |\Psi_1\rangle_i$, where $|\Psi_{23}\rangle_i = \lambda_{i1}|0_20_3\rangle + \lambda_{i2}|0_21_3\rangle + \lambda_{i3}|1_20_3\rangle + \lambda_{i4}|1_21_3\rangle$, $|\Psi_{1'}\rangle_i = \xi_{i1}|0_{1'}\rangle + \xi_{i2}|1_{1'}\rangle$, and $|\Psi_1\rangle_i = \eta_{i1}|0_1\rangle + \eta_{i2}|1_1\rangle$. From the state in Fig. 2(h), further proceeding with one step analogous to that from Fig. 2(b) to Fig. 2(c), we achieve the final state with the originally unpaired Majoranas γ_2^A and γ_3^B braided. This final state can also be decomposed as $|\bar{\Psi}\rangle_f = |\Psi_{23}\rangle_f \otimes |\Psi_{1'}\rangle_f \otimes |\Psi_1\rangle_f$, where $|\Psi_{1'}\rangle_f = \xi_{f1}|0_{1'}\rangle + \xi_{f2}|1_{1'}\rangle$, $|\Psi_1\rangle_f = \eta_{f1}|0_1\rangle + \eta_{f2}|1_1\rangle$, and $|\Psi_{23}\rangle_f$ has the same form as $|\Psi_{23}\rangle_i$, but the λ_{il} are replaced by λ_{fl} , with $l = 1$ to 4. Similarly, the states $|\bar{\Psi}\rangle_i$ and $|\bar{\Psi}\rangle_f$ can also be discriminated using state-tomography techniques.

Discussion and conclusion.—When fabricating superconducting circuits, parameter variations unavoidably occur, as in any solid-state system. For the charge-qubit array, $E_J(\Phi_q) = 0$ can be achieved by having $\Phi_q = \frac{1}{2}\Phi_0$, irrespective of the parameter variations. Also, μ can be tuned, via the gate voltage V_g , to the required value, even if E_{ch} varies for different qubits. For varying E_{J0} among qubits, the interqubit couplings also vary [see Eq. (2)]. One can replace the large Josephson junction by a dc SQUID and tune the SQUID, i.e., the effective E_{Jc} , to

obtain the desired value t for the interqubit coupling. As for the flux-qubit array, $\varepsilon = 0$ can be achieved by having $f = \frac{1}{2}$. Also, μ can be tuned to the given value by changing the flux f_s applied to the SQUID in each qubit. Moreover, even if the parameters of couplers vary, one can tune the flux f_c in each coupler to achieve the required value t for the interqubit coupling [see Eq. (7)].

In conclusion, we propose superconducting circuits to construct two superconducting-qubit arrays where Majorana modes can occur. We suggest using four superconducting qubits as the smallest system to demonstrate the braiding of Majorana modes, and show how to distinguish the states before and after braiding Majorana fermions. These superconducting-qubit arrays can be extended to wire networks, similar to the semiconducting wire networks in [10], to demonstrate the non-Abelian statistics of Majorana fermions.

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